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# Research of omnidirectional mobile robot robust PI control based on H∞ control μ comprehensive method

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**Keywords:** Mecanum wheel; Omnidirectional mobile robot; μ comprehensive robust PI control based on H∞ control; Course Angle deviation; correction

**Abstract:** The omnidirectional mobile robot based on Mecanum wheel is very easy to slide in the course of driving, resulting in the actual course deviation from the set value,in order to improve the accuracy of the omnidirectional mobile robot driving course, an H∞ control μ comprehensive method is proposed for real-time course correction of Omnidirectional mobile robot. Through the 4 steps of H∞ problem standardize, μ comprehensive design, select weight function and D-K iterative algorithm design, the real-time correction of the robot's running direction is realized, and the problems of course deflection and movement instability are solved. The algorithm based on μ comprehensive robust PI control based on H∞ control has good control performance, which satisfies the disturbance suppression performance of nonlinear system and achieves good control effect.

## 1. Introduction

The omnidirectional mobile robot based on Mecanum wheel can achieve basic motion and compound motion such as forward motion, lateral motion, 360° zero radius rotation and so on [1], and can achieve movement and rotation at any direction and at any radius in the plane. Therefore, it is playing an increasingly important role in many fields such as transportation platform, processing platform and modern manufacturing industry [2,3]. Omnidirectional mobile robots are usually composed of four Mecanum wheels in a certain layout, which can achieve Omnidirectional movement at different speeds of each wheel [4]. The movement of Mecanum wheel is easily affected by the ground friction because only one small roller is grounded at one point [5-6], if one of the wheels disengages from the ground and slides, the moving course of the mobile robot will be skewed, resulting in the actual running course and position of the omnidirectional mobile robot deviating from the set course and position of the robot [7].

In order to overcome the problem of Omnidirectional mobile robots, many scholars have studied the method of correcting the position of robots by detecting the Mecanum roller slip momentum. Luo et al. modeled and analyzed the sliding-error caused by unbalanced normal force on Omnidirectional mobile robots, and proposed an online adaptive sliding model based on Mecanum robot to reduce the sliding error [8]. In the field of intelligent control, many researchers suggest that PI control theory can be used to overcome the problem of Mecanum wheel sliding; Tsai et al. proposed a non-singular terminal sliding mode control based on fuzzy wavelet neural network, and used this method to improve the accuracy and stability of trajectory tracking of Omnidirectional mobile robot based on Mecanum wheel [9]. H∞ control method is quite accurate for the design of unstructured uncertain systems, but it is conservative for the design of structurally uncertain systems.

To solve the above problems, In this paper, an μ integrated robust PI control algorithm based on H∞ control is proposed to correct the robot's course. The course accuracy of the mobile robot is guaranteed without any guiding line and special marks.

## 2. Kinematics analysis of omnidirectional mobile robot

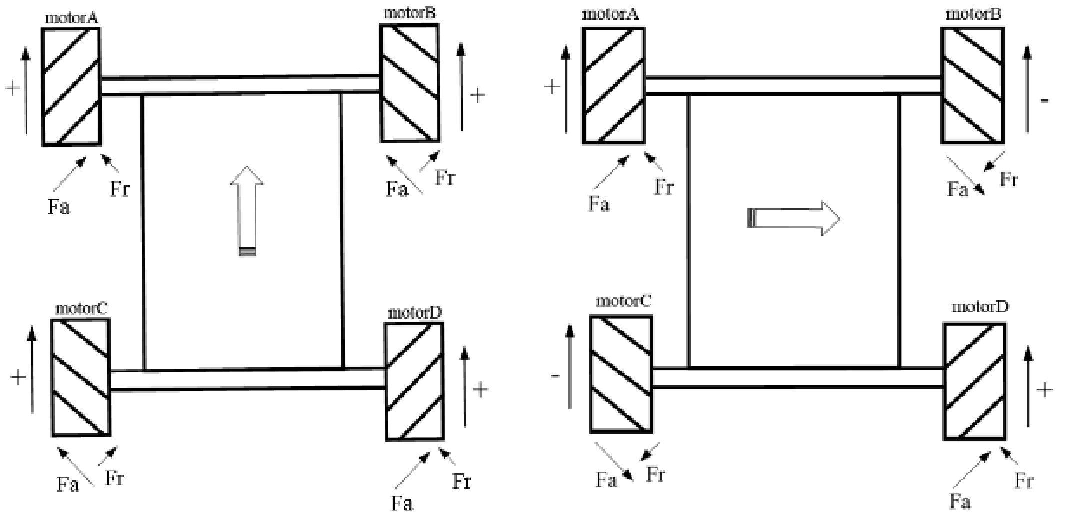
### 2.1 Omnidirectional movement mechanism of Mecanum wheel

In this paper, the Mecanum wheel is designed at an Angle of 45°. The roller can not only revolve around the axis of the wheel and rotate around the axis of the roller, but also rotate around the contact point between the roller and the moving ground, which makes the Mecanum wheel have the motion characteristic of 3 degrees of freedom [10-11].

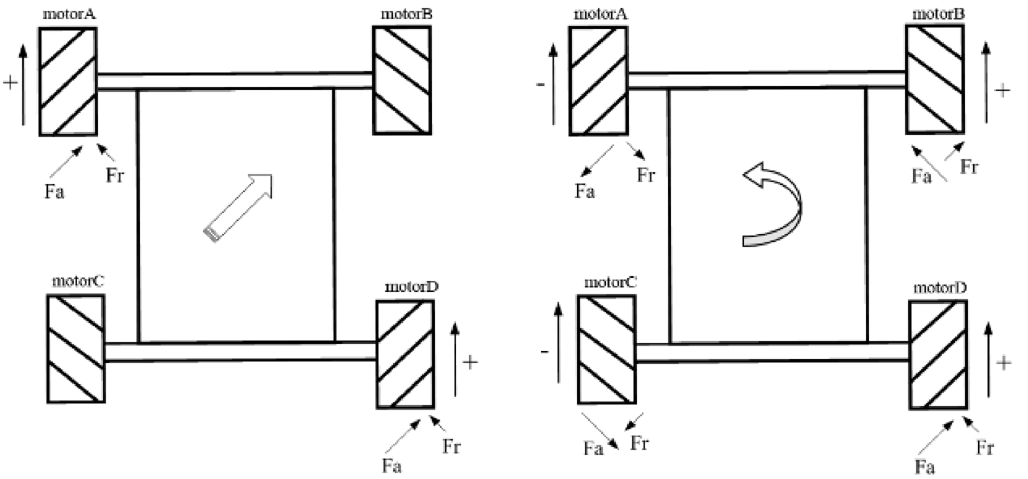
The prominent application of the Mecanum wheel is to make the robot's moving mechanism become omnidirectional by using the wheel to achieve the movement in three directions: horizontal, vertical and zero radius rotation [12]. Figure 1 shows the layout of Mecanum wheel and the analysis of force direction of each wheel in the design of this paper. In the figure, the oblique line of the wheel is the small roller in contact with the ground. Combined with the movement direction and force direction of the wheel, the mechanism of forward, lateral, lateral and rotational motion of the four-wheel mobile robot is demonstrated. Into before, for example, as shown in figure 1 (a), the angular velocity of the four wheels and now the same, small roll axis of each wheel, there is independent of the axial friction, through the comprehensive of axial force of friction, the car of horizontal component force is offset each other, only keep the direction of the force component, so that the mobile robot to move forward.

Fa：Axial friction on a roller

Fr：The roller is subject to rolling friction in the direction of the circumference



(a) Forward motion (b) Lateral motion



(c) Diagonal motion (d) Rotational motion

Figure 1. The force under different motion modes of a robot

## 2.2 kinematics analysis of mobile robot

Figure 2 shows the kinematics analysis model diagram of the mobile robot. The wheel arrangement has a certain influence on the performance of Omnidirectional vehicle [13], In the center of the mobile robot O as the origin of coordinates, to build the global coordinate system XOY, to the rotation of the wheel center $O_i$ as the origin of coordinates, four local coordinate system is set up respectively $X_iO_iY_i$, r is the radius of wheel rotation in the figure, a for car body geometry center O the horizontal distance to the center of rotation for each wheel $O_i$, b for car body geometry center O vertical distance to the center of rotation for each wheel $O_i$, the axis of the roller and wheel axis β Angle is 45 °. The angular velocity of each wheel is $\omega_A$, $\omega_B$, $\omega_C$, $\omega_D$, the counterclockwise rotation of the wheel is positive, while the clockwise rotation is negative, the speed of each roller is $v_{g1}$, $v_{g2}$, $v_{g3}$ and $v_{g4}$ respectively. The transverse velocity, longitudinal velocity and rotation angular velocity of the car are Vx, Vy and ω, respectively. The robot rotates counterclockwise.

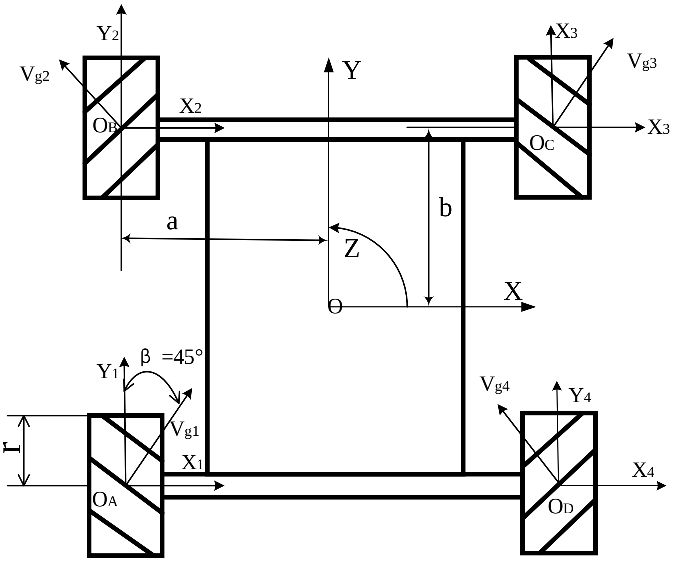


Figure 2. Kinematic analysis model of mobile robot

Taking wheel A as an example, the moving speed of the rotation center OA of the wheel in the global coordinate system XOY is:

$$V_{OA}=\begin{bmatrix} v_x \\ v_y \end{bmatrix}=\begin{bmatrix} 1 & 0 & b \\ 0 & 1 & -a \end{bmatrix}\begin{bmatrix} v_x \\ v_y \\ \omega \end{bmatrix} \tag{1}$$

The moving speed of $O_A$ in the local coordinate system $X_1O_AY_1$ is:

$$V_{OA}=\begin{bmatrix} \mathrm{v_x} \\ \mathrm{v_y} \end{bmatrix}=\begin{bmatrix} 0 & \sin\beta \\ \mathrm{r} & \cos\beta \end{bmatrix}\begin{bmatrix} \omega_A \\ \mathrm{v_{g1}} \end{bmatrix} \tag{2}$$

From formula (1) and (2):

$$V_{OA}=\begin{bmatrix} 0 & \sin\beta \\ \mathrm{r} & \cos\beta \end{bmatrix}\begin{bmatrix} \omega_A \\ \mathrm{v_{g1}} \end{bmatrix}=\begin{bmatrix} 1 & 0 & b \\ 0 & 1 & -a \end{bmatrix}\begin{bmatrix} v_x \\ v_y \\ \omega \end{bmatrix} \tag{3}$$

obtained:

$$\omega_A=\begin{bmatrix} -\dfrac{1}{\mathrm{r}\tan\beta} & \dfrac{1}{\mathrm{r}} & -\dfrac{a\tan\beta+b}{r\tan\beta} \end{bmatrix}\begin{bmatrix} v_x \\ v_y \\ \omega \end{bmatrix} \tag{4}$$

Similarly, kinematics analysis of the rest of the wheels can be obtained as follows:

$$\begin{bmatrix} \omega_A \\ \omega_B \\ \omega_C \\ \omega_D \end{bmatrix} = \begin{bmatrix} -\frac{1}{\mathrm{r}\tan\beta} & \frac{1}{\mathrm{r}} & -\frac{a\tan\beta+b}{r\tan\beta} \\ \frac{1}{\mathrm{r}\tan\beta} & \frac{1}{\mathrm{r}} & -\frac{a\tan\beta+b}{r\tan\beta} \\ -\frac{1}{\mathrm{r}\tan\beta} & \frac{1}{\mathrm{r}} & \frac{a\tan\beta+b}{r\tan\beta} \\ \frac{1}{\mathrm{r}\tan\beta} & \frac{1}{\mathrm{r}} & \frac{a\tan\beta+b}{r\tan\beta} \end{bmatrix} \begin{bmatrix} \mathrm{v}_x \\ \mathrm{v}_y \\ \omega \end{bmatrix} \tag{5}$$

$$= J \begin{bmatrix} \mathrm{V}_x \\ \mathrm{V}_y \\ \omega \end{bmatrix}$$

Where J is the Jacobian matrix of the inverse kinematics of the system.

According to the inverse kinematics principle, when the Jacobian matrix of the inverse kinematics of the system is singular matrix, the degree of freedom of the system will decrease. When the Jacobian matrix of the inverse kinematics of a moving system is nonsingular, it has the ability to move in all directions. By comparing with the matrix J in formula (5), it can be seen that rank (J) =3, so the system designed in this paper has the ability of omnidirectional movement.

## 3. PI control algorithm description of μ comprehensive robustness based on H∞ control

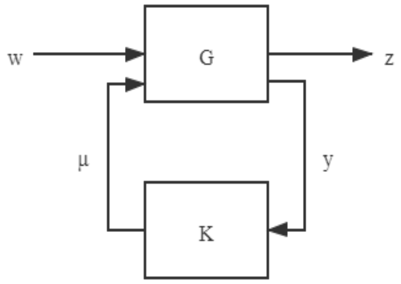
### 3.1 μ integrated H∞ controller design

The key of μ comprehensive robust PI control algorithm based on H∞ control is the design of μ comprehensive H∞ controller. The design process of μ comprehensive H∞ controller proposed in this paper can be divided into four steps: H∞ problem standardize, μ comprehensive desig, select weight function and D-K iterative algorithm design.

### 3.2 H∞ problem standardize

Various H∞ control problems can be converted into the standard problems shown in figure 3. In the figure, w,z, j and y are vector-valued signals: w is a one-dimensional external input signal, which generally includes instruction (reference) signal, interference and sensor noise. z is the controlled output of P dimension, which usually includes tracking error, adjustment error and actuator output. y is the output of m-dimension measurement, which may be the output of the sensor and the instruction signal. In the figure, G and K are respectively generalized controlled objects and controllers. The controller K needs to be designed. It is assumed that G and K are frequency domain descriptions of linear time-invariant systems and that the transfer function matrices G(s) and K(s) are true rational function matrices.

The basic block diagram of standard H∞ control problem is shown in figure 3 below:



G——Controlled object; K——The controller;
w——External input; μ——Control input;
z——Accused of output; y——Measured output

Figure 3. basic block diagram of standard H∞ control problem

Suppose G and K are the transfer function matrix descriptions for linear time invariant systems. The generalized G(s) of the controlled object can be expressed as formula (6):

$$G(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{13}(s) & G_{14}(s) \end{bmatrix} \tag{6}$$

The closed-loop transfer function from W to Z can be expressed as formula (7):

$$(T_{zw}s) = F_l(G,K) = G_{11} + G_{12}K(I - G_{22}K)^{-1}G_{21} \tag{7}$$

The standard problem of H∞ control is to find A physically realizable controller K, to make the closed-loop system stable, and to make the H∞ norm of the transfer function matrix Fl (G, K) minimal, that is $\min\|\mathrm{F}_l(G,K)\|_\infty$.

### 3.3 μ comprehensive design

The standard structure of μ comprehensive is shown in FIG. 4, where P is the generalized controlled object, K is the controller to be designed, and Δ is the disturbance block. Through the transformation, it can be transformed into the structure shown in figure 5 (the same as figure 3), Where M= $F_u$ (P, Δ), it can be seen from the control diagram of H∞ that the comprehensive problem of μ has been transformed into the design problem of H∞, but at this time, the generalized controlled object M contains an uncertain disturbance block Δ, that is, the disturbance suffered in the running.

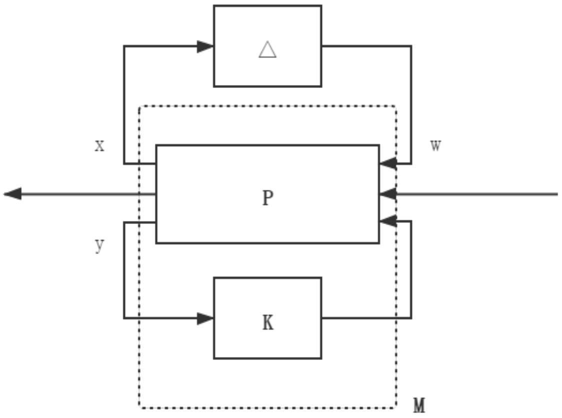


Figure 4. shows the comprehensive standard structure diagram

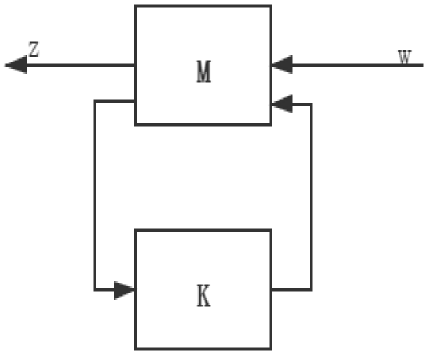


Figure 5. the generalization schematic diagram represented by H∞

Where, μ the purpose of the comprehensive design method is to seek A physically achievable controller K, so that the H∞ norm of the transfer function from w to z reaches $\min\|\mathrm{F}_l(G,K)\|_\infty$. Such designed controller K disturbance Δ can ensure the existence circumstances the robust stability of closed-loop system, because in the design process has considered the system performance requirements (reflect on the choice of weight function), therefore, to design the controller K to ensure that the system has good dynamic and static performance and anti-interference performance, experiments choose μ comprehensive H∞ controller can completely meet the requirements of the system as a whole.

When H∞ and μ comprehensive methods are adopted to design the controller, the system can be first converted into the standard form of μ comprehensive structure as shown in FIG. 5, and then the μ comprehensive and analysis toolbox in MATLAB is used to complete the design of the controller [14].

### 3.4 Select weight function

According to H∞ control theory, three weighting functions $W_1(s)$、$W_2(s)$、$W_3(s)$ are needed to transform the nominal controlled object model into H∞ standard design problem [15]. In view of the parameter uncertainty and load disturbance of the system, the stability and anti-disturbance of the system are the main objectives of the design, that is, the hybrid robust control problem of stability and quality [16].

The structure block diagram of the augmented controlled object composed of the nominal controlled object and the weighting function is shown in FIG. 6:

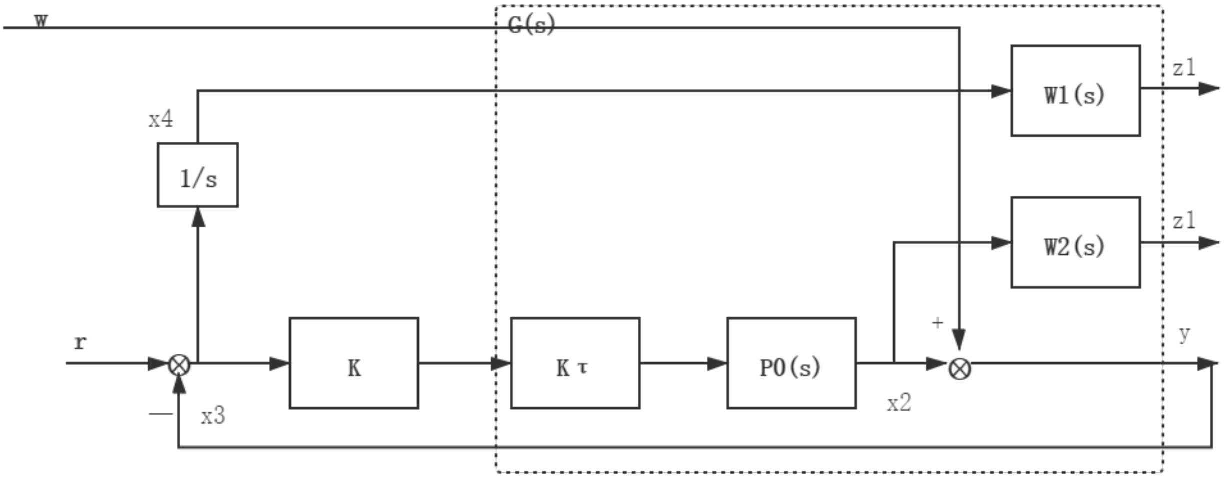


Figure 6. enlarges the structure diagram of the controlled object

The graph shows the state variables and where the weighted function is added. Where: $W$ is the interference input signal; $Z_1$ and $Z_2$ are the evaluation signals defined in response to the design requirements, namely overshoot and overshoot time; $x_1$、$x_2$、$x_3$、$x_4$ is the state variable.

### 3.5 D-K iterative algorithm design

Iterative computation is a typical method in numerical computation, which is applied to the roots of equations, the solution of equations, and the eigenvalues of matrices [17]. Iteration is an activity that repeats the feedback process, usually in order to approximate the desired goal or outcome [18].

The basic idea of iterative computation is successive approximation, first take a rough approximation value, and then use the same recursive formula to repeatedly correct the initial value until the predetermined accuracy is achieved. The variation trend of each variable in the iterative algorithm is shown in figure 7:

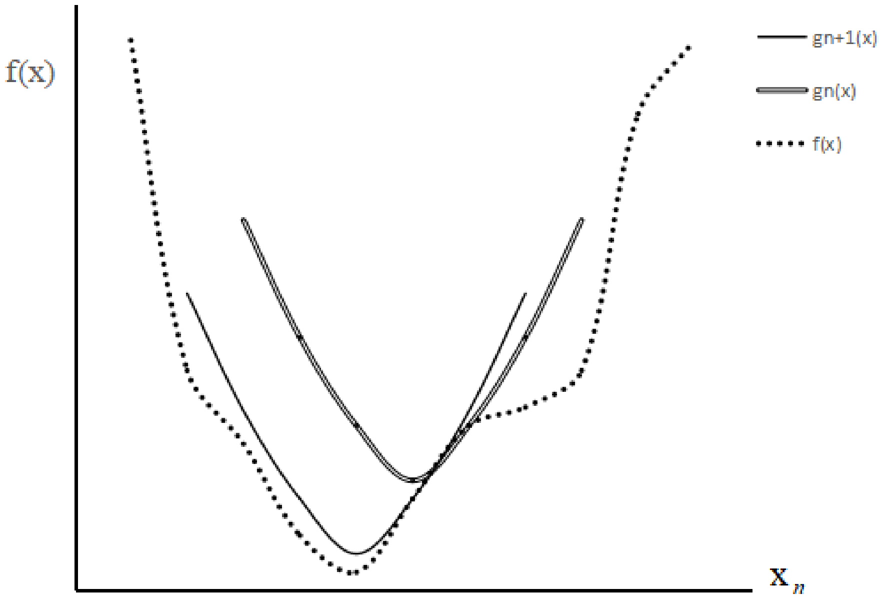


Figure 7. variation trend of each variable used in the iterative algorithm

## 4. The application of robot speed control system based on H∞ control μ comprehensive method

The block diagram of the speed regulation control system of the robot is shown in FIG. 8, the values of value and Δvalue are given to the CCRx register of STM32F103 advanced timer. Assigning different values to the CCRx register can generate different PWM output waves to control the steering and speed of the dc motor [19]. When there is no feedback adjustment of navigation Angle, the motor speed is precisely controlled by the value value output from the traditional speed PI control algorithm. When the robot's moving speed is set, the target speed of each motor is calculated by formula (5). The actual speed of the motor is fed back by the encoder to find out the deviation e between the target speed of the motor and the actual speed, speed PI controller calculates the value of the value through deviation e, then generates PWM wave to achieve accurate control of the motor speed.

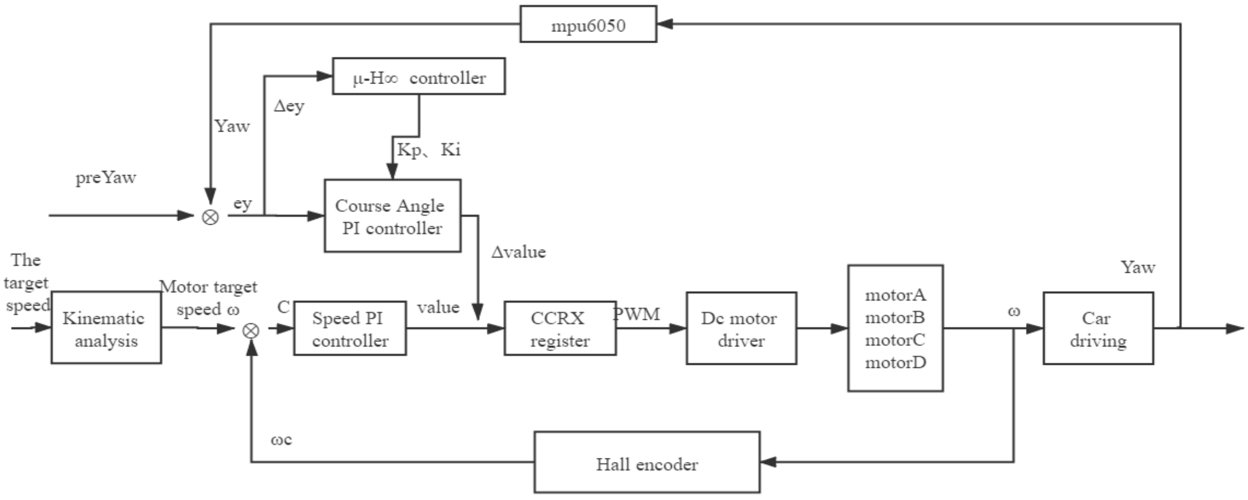


Figure 8. block diagram of speed control system for robot

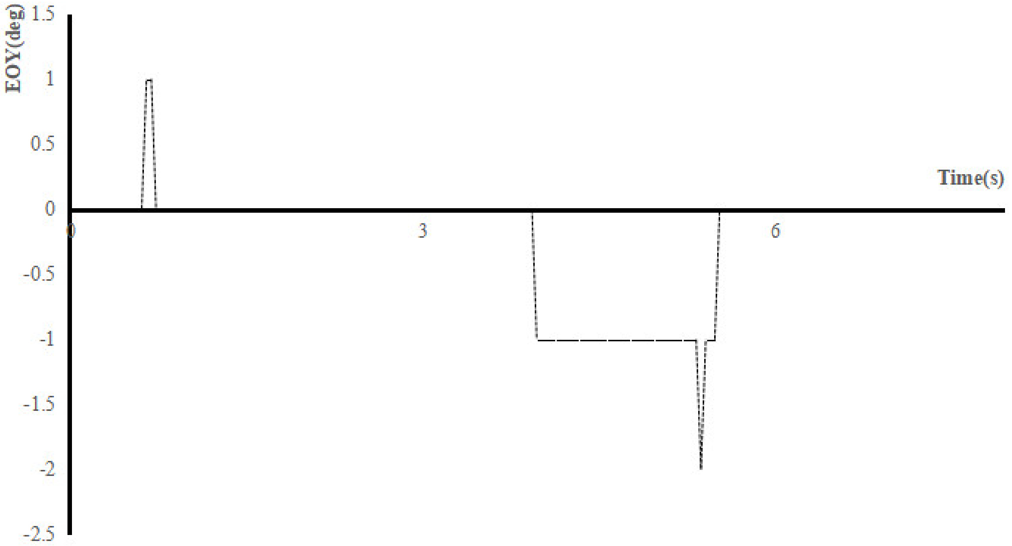
The algorithm of course correction of robot based on the feedback course Angle adopts A. As shown in FIG. 8, controller A is composed of the traditional course Angle PI controller and A. It does not use the PI adjustment algorithm with fixed parameters in the control, Instead, according to different situations, PI parameters are adjusted in real time by controller A to adjust these delay control schemes, so that the closed-loop system is stable [20]. The inertial sensor mpu6050 provides real-time feedback to the robot's course, when the actual course of the robot deviates from the set course, The control system will detect the deviation of the robot's course Angle $e_y$($e_y$=Yaw-PreYaw) and the increment of course Angle deviation $\Delta e_y$=($e_y$-Last_$e_y$), In addition, $e_y$ and $\Delta e_y$ are sent to the μ comprehensive method based on H∞ control controller to dynamically adjust the two parameters Kp and Ki of the μ comprehensive method based on H∞ control in real time.

The basic expression of the μ comprehensive method based on H∞ control is:

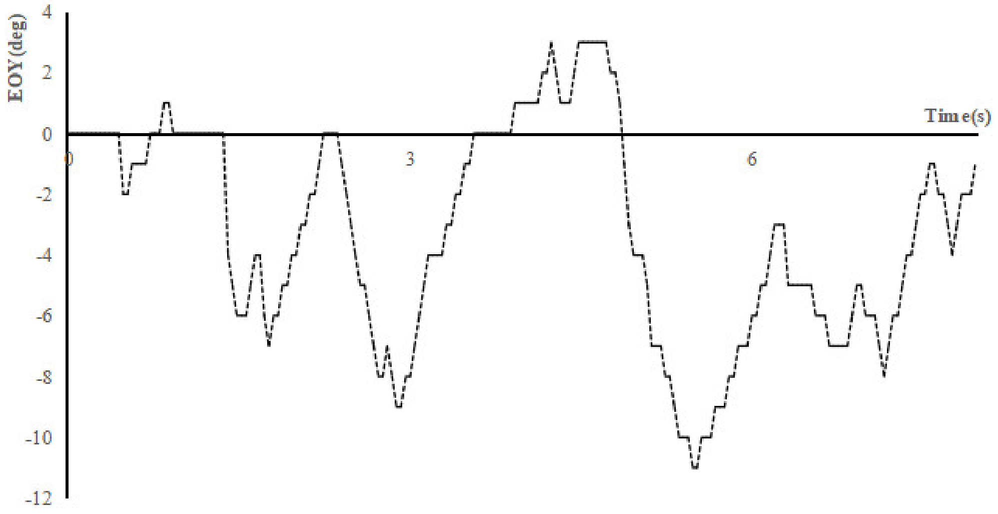
$$\Delta value = K_p \times \Delta e_y + K_i \times e_y \tag{8}$$

Course Angle PI controller through formula (8) produce Δvalue to adjust the specified rotational speed of every motor of the robot, then correct the robot's movement course. When the correction reaches the set course of the robot, Δvalue will automatically become 0. At this time, the motors will return to the original target speed according to the traditional speed PI control algorithm to complete the automatic accurate correction.

For the above omnidirectional mobile robot system, we get the data graph as shown in FIG. 9(a) and 9(b) through the simulation of different control methods.



(a) Heading deviation of the μ comprehensive method based on H∞ control under large disturbance



(b) Heading deviation of traditional PI control under large disturbance

Figure 9. Comparison diagram of feedback correction between traditional PI control and the μ comprehensive method based on H∞ control under large disturbance during longitudinal driving

## 5. Conclusion

Aiming at the problem of course deviation caused by the sliding of Mecanum wheel during the movement of omnidirectional mobile robot based on Mecanum wheel, This paper proposes a course correction method for robot motion based on the course Angle of inertia sensor feedback with the μ comprehensive method based on H∞ control.

1) Using the μ comprehensive method based on H∞ control, the PI control of the Omnidirectional mobile robot is realized, and the problem of course deviation caused by Mecanum wheel sliding is solved theoretically, which improves the transport stability of the Omnidirectional mobile robot.

2) Through the analysis of the advantages and disadvantages between the traditional PI control algorithm and the μ comprehensive method based on H∞ control in the direction adjustment performance of the robot, it can be seen that the μ comprehensive method based on H∞ control has better dynamic performance and stability than the traditional PI control system, which can not only reduce the oscillation, but also adjust faster, achieving the effect of faster response speed and more accurate control.

3) The closed-loop system has good dynamic, static and anti-interference performance.

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